## Homework # 5

DUE THURSDAY, NOVEMBER 30, 2000, AT 2:30 PM

*Collaboration in the sense of discussions is allowed, but you should write the final solutions alone and understand them fully. Do not read class notes or homework solutions from previous years at any time. Other books and notes can be consulted, but not copied from. You should justify your answers, at least briefly. Definitions and notation follow the lectures.*
*The handouts and data for the homeworks can be found at:*

**http://work.caltech.edu/cs156/00/homeworks.htm**

**1. Support Vectors - Geometrically**

Consider the nonlinear transformation of the input space $X$ in problem **1** of HW# 4 into another two-dimensional space $Z$

$$z_1 = x_2^2 - 2x_1 - 1 \qquad z_2 = x_1^2 - 2x_2 + 1$$

(i) Transform the training set of problem **1** into the $Z$ space.

(ii) Find the optimal separating 'hyperplane' in the $Z$ space, and identify the support vectors.

(iii) Plot the corresponding decision boundaries in the $X$ space, and identify the support vectors.
(Remark: The nonlinear transformation needs to be fixed before the training set is known *to guarantee the VC bounds*)

**2. Support Vectors - Analytically**

Consider the same training set of problem 1, but instead of explicitly transforming the input space $X$, apply the SVM algorithm with the dot product

$$K(\mathbf{x}, \mathbf{x}') = (1 + \mathbf{x}.\mathbf{x}')^2$$

(which corresponds to a second-order polynomial transformation)

(i) Set up the expression for $\mathcal{L}(\lambda_1 ... \lambda_7)$ and solve for the optimal $\lambda_1, ..., \lambda_7$ (numerically).

(ii) Plot the decision boundaries in the $X$ space, and identify the support vectors.

**3. More Support Vectors**

Repeat problem 2 after adding the point $\mathbf{x}_8 = (0, 0)$ , $y_8 = +1$ to the training set. Use the lowest-order polynomial that would work, i.e., the smallest $M$ for which the dot product

$$K(\mathbf{x}, \mathbf{x}') = (1 + \mathbf{x}.\mathbf{x}')^M$$

would result in a SVM with zero training error.

**4. Jeopardy!**

Write your own problem on any part of the course. Either provide a solution for it, an outline of a solution, or the reasons why a solution is difficult or tricky.