Due Friday February 23 at 4pm at the electronic server or in the box in the VLSI lab

1. **(**10 **points)** Show how to efficiently construct the level graph L_G of a capacitated graph $G = (V, E, c)$ out of a given source vertex $s \in V$.

2. **(**10 **points)** Suppose that $G = (V, E, c)$ a capacitated graph in which all capacities are 0 or 1. Show how a max flow between a given source s and sink t can be computed in time $O(nm)$. (Here $n = |V|$ and $m = |E|$.)

3. **(**10 **points)** Prove the König-Egerváry theorem:

   Let A be a matrix containing 1's and 0's.

   A "matching" of size k in A is a set of pairs $\{i_1, j_1\}, ...\{i_k, j_k\}$ such that all the $\{i_\ell\}$ are distinct, all the $\{j_\ell\}$ are distinct, and $A_{i_\ell, j_\ell} = 1 \; \forall \ell$.

   If R is a set of rows of A and C is a set of columns of A, then $R \cup C$ is a "cover" if every entry $\{i, j\}$ for which $A_{i,j} = 1$, lies in either a row of R or a column of C. The size of the cover is $|R| + |C|$.

   Theorem: the least size of a cover is equal to the maximum size of a matching.

   Hint: Consider A as the adjacency matrix of a bipartite graph. Relate the question to matchings in this graph, and in turn, to flows and cuts in a related capacitated graph.

4. **(**5 **points)** CLR 27.2-8.

5. **(**5 **points)** CLR 27.2-9.

6. **(**10 **points)** CLR 27-4.