HW3 Solution

1 Problem 1

Sol: Without inverting two bits, the all-low gross error would be interpreted as the all-zero codeword; we cannot invert only 1 bit since then the all-low gross error would be recognized as a single error corruption of the all-zero codeword and then would be corrected to the all-zero codeword. On the other hand, inverting two bits will guarantee that both all-low and all-high gross errors would be detected but not corrected, which is what we want.

2 Problem 2

Sol: a) Use the sphere bound:

$$2^k V_2(n, 2) \leq 2^n. \tag{1}$$

A little manipulation gives us:

$$2^{r+1} \geq (k + r)^2 + (k + r) + 2. \tag{2}$$

b) For a $(n, k, 6)$ code, if we delete the first bit of each codeword, we obtain a $(n - 1, k, \geq 5)$ code. Hence, by the sphere bound, we have:

$$2^k V_2(n - 1, 2) \leq 2^{n-1}. \tag{3}$$

So,

$$2^r \geq (k + r)^2 - (k + r) + 2. \tag{4}$$

c) For $k = 32$, if $d = 5$, $r \geq 10$; if $d = 6$, $r \geq 11$.

3 Problem 3

Remark: Everyone figured out that one should prove each non-zero codeword has weight 2^{m-1}, but not everyone succeeded. There are several good ways to prove this fact. In the following I give out what I think two best ways.
1) (submitted by Ling Li.) First notice columns of G are all distinct nonzero

vertos of length m. For a message $x \neq 0$, there is at least one column vector g such that $xg = 1$. Then we pair all column vectors of G other than g by the following way,

$$g^{'} \leftrightarrow g^{'} + g. \tag{5}$$

Clearly if $xg^{'} = 1$, then $x(g^{'} + g) = 0$, and vice versa. So the weight of xG is

$$1 + \frac{(2^m - 1) - 1}{2} = 2^{m-1}. \tag{6}$$

2) For a message $x = (x_1, x_2, \ldots, x_m) \neq 0$, assume that i_0 is the first i such that $x_i = 1$. Now we pair up all column vectors of G by the following way: g_1 *and* g_2 *make a pair if and only if they are the same except at position* i_0. In such pairing, only one vector, namely $g_0 = (0, \ldots, 1, 0, \ldots, 0)$ where the 1 is at i_0, is left out. Clearly for a pair g_1 and g_2, $xg_1 + xg_2 = 1$. Since $xg_0 = 1$, we have the weight of xG is:

$$1 + \frac{(2^m - 1) - 1}{2} = 2^{m-1}. \tag{7}$$

Hence,

$$A(z) = 1 + (2^m - 1)z^{2^m - 1}. \tag{8}$$

4 Problem 4

Sol: Plug into MacWilliams identity, simplify a little, it's straightforward to obtain:

$$B_3 = \frac{(2^m - 1)(2^{m-1} - 1)}{3}, \tag{9}$$

and

$$B_4 = \frac{(2^m - 1)(2^{m-1} - 1)(2^{m-2} - 1)}{3}. \tag{10}$$

5 Problem 5

Sol: $10^6 mod 168 = 64$, which amounts to 2 days and 16 hours. For simplicity, let's represent this by $(2, 16)$. And we represent Wednesday noon by $(3, 12)$. a) $(3, 12) + (2, 16) = (6, 4)$, so its Saturday 4 am. b) $(3, 12) - (2, 16) = (0, 20)$, so its Sunday 8 pm.