EE/Ma 127a Error-Correcting Codes — R. J. McEliece
draft of October 11, 2000 — 162 Moore

Homework Assignment 2 (**Final Version**)
Due (in class) 9am October 13, 2000

Reading: Wicker, Chapter 4, Section 4.1 (pp. 69–81).

Problems to Hand In:

Problem 1. Desribe the parity-check matrix, and an appropriate decoding algorithm, for an $(n, 32)$ binary linear code that is capable of detecting all error patterns of weight ≤ 3, with n as small as possible.

Problem 2. Wicker, Problem 4.8 (p. 97) parts 9(a) and (b) only. Note: The minimum distance of a linear code is the same as the minimum weight among all (nonzero) codewords.

Problem 3. In class on Oct. 9, I showed that for the binary symmetric channel, *maximum likelihood* decoding (i.e., find the codeword $\boldsymbol{x}_i$ for which $p(\boldsymbol{y}|\boldsymbol{x}_i)$ is largest) is the same as *minimum (Hamming) distance* decoding (i.e., find the codeword $\boldsymbol{x}_i$ for which $d_H(\boldsymbol{x}_i, \boldsymbol{y})$ is smallest). In this problem you are supposed to find a similar simplification of ML decoding for two other channel models: the *binary erasure channel* and the *Z-channel.* The input-output transition probabilities for the channels are as follows, where p is a number between 0 and 1/2.

(a) The binary erasure channel:

$$\begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & ? \end{array} \\ \begin{pmatrix} 1-p & 0 & p \\ 0 & 1-p & p \end{pmatrix} \end{array}.$$

(b) The Z-channel:

$$\begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{pmatrix} 1 & 0 \\ p & 1-p \end{pmatrix} \end{array}.$$

Problem 4. Consider the $(8, 4)$ binary linear code described in Homework Assignment 1, Problem 2.

(a) What is the minimum distance of the code?

(b) Suppose the code is used with a bounded distance decoder, as described in class on October 11, with $t = 0$, and the channel is a binary symmetric channel with crossover probability p. As a function of p, what is the probabiity of decoder *error*?