EE/Ma 127a Error-Correcting Codes — R. J. McEliece

draft of October 2, 2000 — 162 Moore

Homework Assignment 1 (Final version)

Due (in class) 9am October 4, 2000

Reading: Wicker, Chapter 1, Sections 1.1–1.3.
Chapter 4, Section 4.1 (pp. 69–72), Sec. 4.2, 4.3, and Sec. 4.5.

Problems to Hand In:

Problem 1. In class on September 25 I defined the $(7,4)$ Hamming code via a Venn diagram with seven compartments, and demonstrated that it was capable of correcting any single error, or any double erasure.

Of the $\binom{7}{3} = 35$ patterns of three erasures, which (if any) is the Hamming code capable of correcting?

Problem 2. Find a basis for the nullspace (sometimes caled the "orthogonal complement") for the $(8,4)$ binary linear code with generator matrix

$$G = \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Problem 3. Let

$$G = \begin{pmatrix} 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \end{pmatrix}$$

be a generator matrix for a $(6,3)$ binary linear code C.

(a) Find a row-reduced echelon generator matrix for C.

(b) Find a parity-check matrix for C.

Problem 4. In class on Friday 9/29 I defined the Gaussian q-binomial coefficient

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \prod_{i=0}^{k-1} \frac{q^n - q^i}{q^k - q^i},$$

(valid for $n \geq k \geq 1$) and showed that the number of (n,k) binary linear codes is $\begin{bmatrix} n \\ k \end{bmatrix}_2$.

(a) Find the number of $(7,k)$ binary linear codes for $k = 1, 2, \ldots, 7$.

(b) Assuming that q is a real number, evaluate the limit

$$\lim_{q \to 1} \begin{bmatrix} n \\ k \end{bmatrix}_q.$$

Problem 5. Suppose you wanted to design an (n, k) linear block code, with $k = 32$, capable of correcting all single bit errors and detecting all double bit errors.

(a) What is the smallest value of n for which this is possible?

(b) Describe a parity-check matrix for the code.