CNS/CS/EE 188a Computation Theory and Neural Systems - Homework 6.2 (b)

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Sorting. For n even and inputs $x_1, x_2, \ldots, x_n$, construct n integers with n bits each:

$$\begin{array}{rcccc}
X_1: & x_1 & x_2 & 1 & \mathbf{0} \\
X_2: & x_3 & x_4 & 1 & \mathbf{0} \\
 & \vdots & & & \\
X_{\frac{n}{2}}: & x_{n-1} & x_n & 1 & \mathbf{0} \\
X_{\frac{n}{2}+1}: & 1 & 1 & 0 & \mathbf{1} \\
X_{\frac{n}{2}+2}: & 1 & 1 & 0 & \mathbf{2} \\
 & \vdots & & & \\
X_n: & 1 & 1 & 0 & \boldsymbol{\frac{n}{2}}
\end{array}$$

Here I use bold symbol to represent the binary form of a number (and, of course, with 0's padded in the front of the binary string). For example, $\mathbf{5} = 0\ldots0101$. For $i \leq \frac{n}{2}$ and $j > \frac{n}{2}$, $X_i \neq X_j$; and $X_i > X_j$ iff $x_{2i-1} \wedge x_{2i}$ is 1; and for $j_1 > j_2 > \frac{n}{2}$, $X_{j_1} > X_{j_2}$.

Sort these n numbers and we get n sorted numbers $X'_1, X'_2, \ldots, X'_n$. Assume the sorting network makes $X'_1 \leq X'_2 \leq \cdots \leq X'_n$. Then we claim that $IP2(x_1, x_2, \ldots, x_n) = X'_{\frac{n}{2},0}$, the least significant bit of $X'_{\frac{n}{2}}$. Proof omitted.

Note: In order to accommodate $\frac{n}{2}$ in $n-3$ bits, we need

$$2^{n-3} > \frac{n}{2}.$$

That is, $n \geq 6$ (since n is even).