CNS 187 Neural Computation - Problem Sheet 2

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2.1 Integrate & Fire Neurons

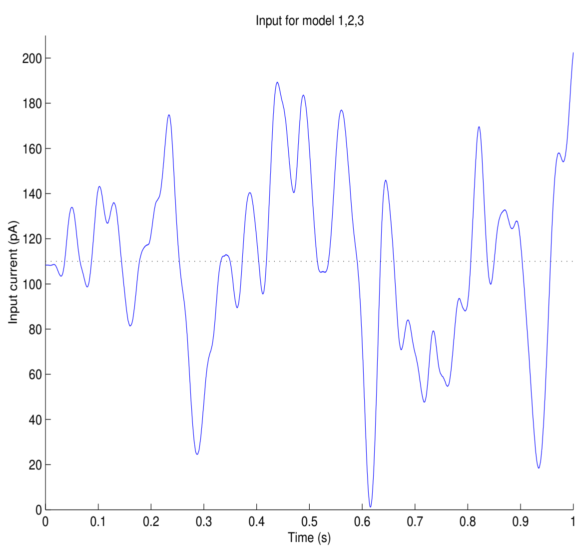
1. Three type of input currents are used here:

   (a) constant current, range from 0 to 500pA;

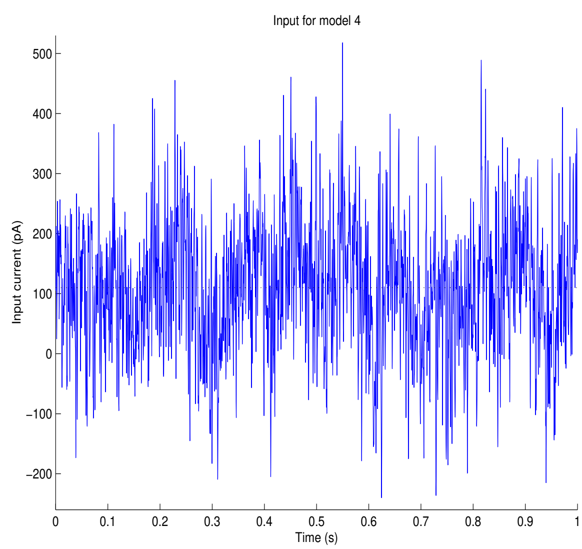
   (b) white noise, bandwidth = 10Hz, mean = 0, standard deviation $\sigma = 40$pA;

   (c) another white noise, bandwidth = 300Hz, mean = 0, $\sigma = 100$pA.

2. We use a constant current 1a of 110pA plus 1b and/or 1c (Figure 1) as input current to the neurons. Simulating with time step of 0.2ms, we get the membrane voltage and spikes in



(a) Current 1a (110mA) plus 1b

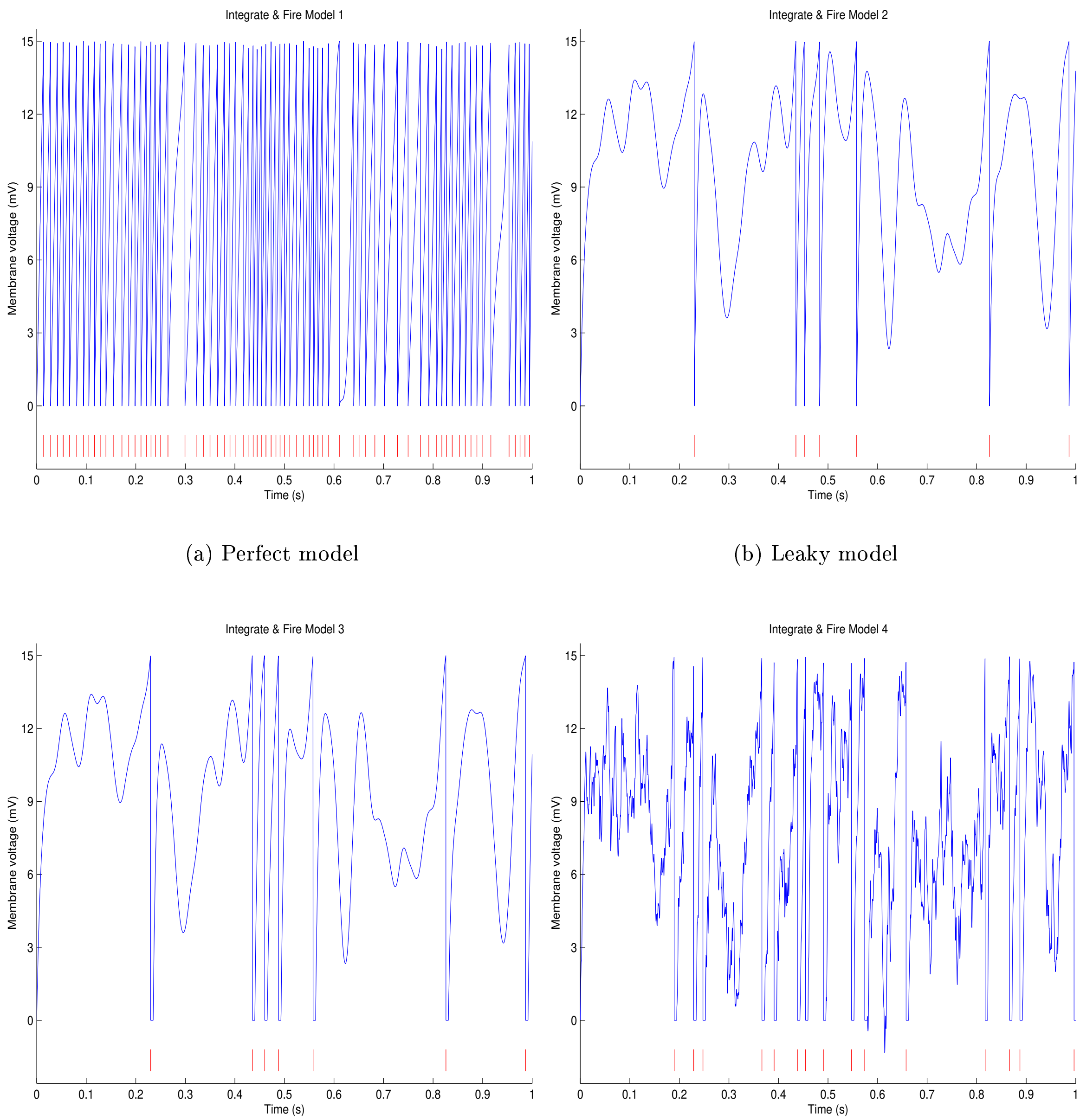


(b) Adding 1c to current in subfigure (a)

Figure 1: Input currents

   Figure 2 on page 2.

3. Remove white-noise 1b from the inputs to all models and vary the constant current, we get the spiking frequency v.s. input current in Figure 3. Below is the discussion:



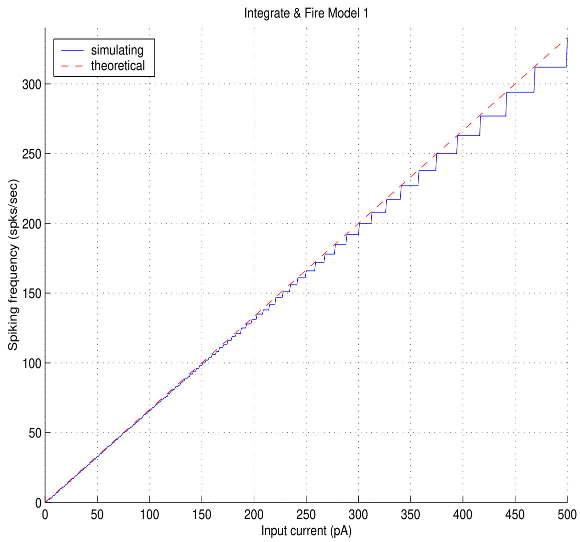
(a) Perfect model

(b) Leaky model

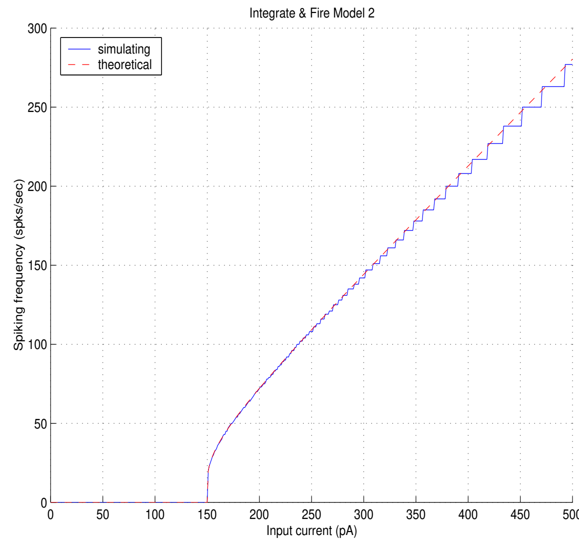
(c) Refactory model

(d) Refactory model with extra noise

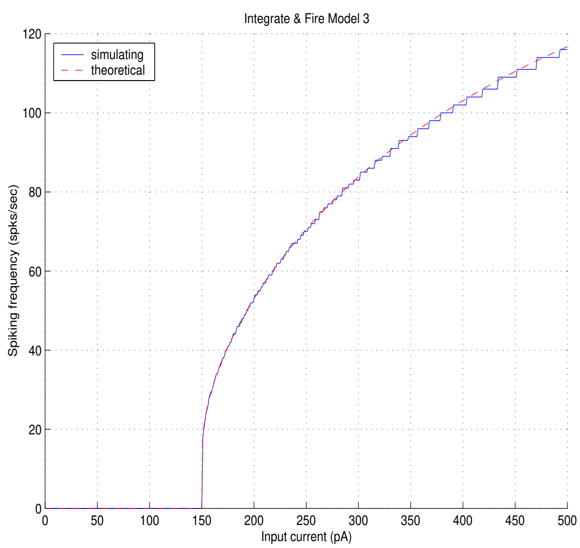
Figure 2: Membrane voltage and spikes, using input from Figure 1. (simulating step 0.2ms) Parameters for these four models are: $C = 100\text{pF}$, $V_t = 15\text{mV}$, and (a) $R = \infty$, $T_{\text{ref}} = 0$; (b) $R = 100\text{M}\Omega$, $T_{\text{ref}} = 0$; (c,d) $R = 100\text{M}\Omega$, $T_{\text{ref}} = 5\text{ms}$.



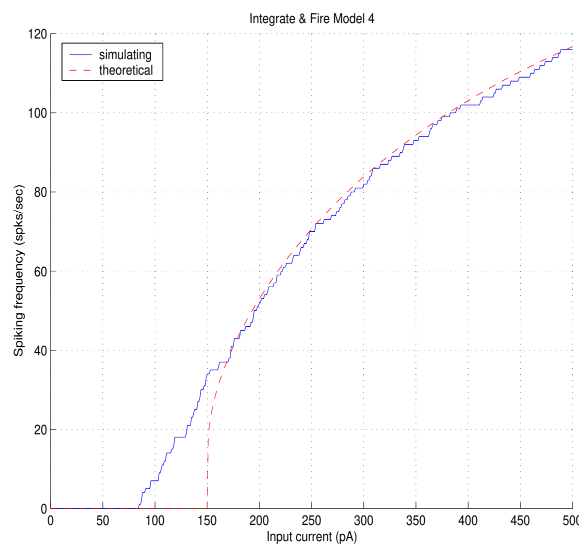
(a) Perfect model



(b) Leaky model



(c) Refactory model



(d) Refactory model with extra noise

Figure 3: Spiking frequency v.s. input current (simulating step 0.2ms). Parameters for models can be found in Figure 2. Due to simulation error, there're steps in simulating curves. The theoretical curves are also shown.

(a) Threshold and theoretical frequency. From the curcuit equation,

$$C\frac{dV}{dt} = -\frac{V}{R} + I, \tag{1}$$

the threshold input current is $I_t = \frac{V_t}{R}$, below which there will be no spikes. For $R = 100\text{M}\Omega$ and $V_t = 15\text{mV}$, we have $I_t = 150\text{pA}$. This is shown in Figure 3(b) and 3(c). Also from (3), the theoretical frequency is (when $I > I_t$)

$$F = \frac{1}{T_{\text{ref}} - RC\ln(1 - V_t/IR)}.$$

The theoretical curves are also plotted in Figure 3 for comparison.

(b) Simulation error. We can see steps in the simulating curves in Figure 3(a), 3(b) and 3(c). A typical spiking interval of model 2 when $I = 450\text{pA}$ is about 4ms. If we use 1ms as the simulating time step, which is in the same order of magnitude as the spiking interval, we will get Figure 5 on page 8, with even larger steps in curves. For Figure 3, a smaller time step (0.2ms) is used. However, this can not eliminate the steps.

(c) Noise gives chance to an early spiking. Look at Figure 3(d). White noise 1c adds variation to the constant current, and sometimes makes the input current bigger than I_t while the constant current itself is below I_t.

4. Adder.

   (a) **Simulating**. Use the network (of 4 neurons) I designed in Problem Sheet 1, I implemented the simulation by use w_{ij} in the Matlab code. I found out that the variance of input current (due to different input patterns to the neurons) will make the spiking happen before or later than what I want them to be.

   (b) **Adding delay gates**. Adding a delay gates after each neuron's output is a good way to synchronize. We can introduce a clock signal. To insert a delay gate after a neuron, say neuron α, we use a neuron with two inputs, one is the clock signal and the other is α's output, and setting the threshold of the delay neuron to 2. I prefer to make at most one spiking during Δt time, so there're some rules to obey

   $$T_{\text{ref}} \le \Delta t - T_\theta \le d$$

   where $T_\theta + T_{\text{ref}}$ is the spiking interval, and d is the maintaining time in synaptic current.

2.2 Network Equivalences

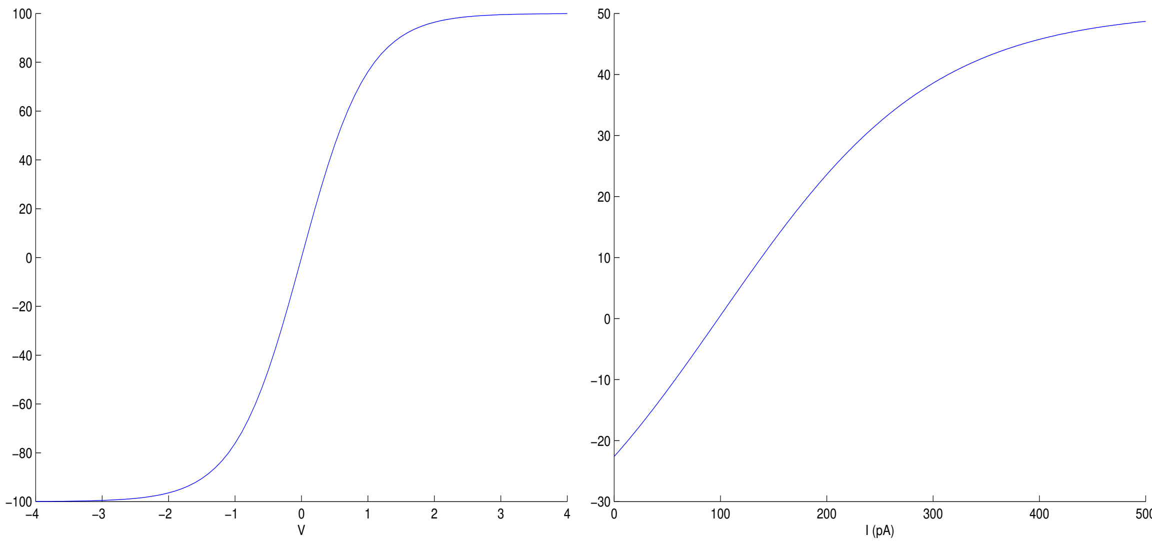
1. Suppose δ and Δ in

   $$g(x) = 2\sigma(\delta x + \Delta) - 1 \tag{2}$$

   is known and $\delta > 0$. This is practical since g and σ both shoud increase when x inceases. A sketch of g and σ can be found in Figure 4.
   Below are lots of equations. The results are (4), (8), (9), (11) and (12).

   (a) Plug $V_i = \alpha I_i + \beta$ into (2) with $x = V_i$, we have

   $$g(V_i) = 2\sigma(\delta\alpha I_i + \delta\beta + \Delta) - 1.$$



(a) $g(V) = 100\tanh(V)$ (b) $\sigma(I)$

Figure 4: g and σ, $\delta = 2 \times 10^{-10}$, $\Delta = 100\text{pA}$

Since we do not know the exact form of σ and g, we should make the variable of σ here and that in

$$\frac{dI_i}{dt} = -\frac{1}{\tau_i} I_i + \sum_{j=1}^{N} w_{ij}\sigma(I_j) + I_i^{\text{in}} \tag{3}$$

just the same. Let $\delta\alpha I_i + \delta\beta + \Delta = I_i$, we get

$$\alpha = \delta^{-1}, \beta = -\delta^{-1}\Delta \tag{4}$$

(b) Plug $g(V_i) = 2\sigma(I_i) - 1$ and $V_i = \alpha I_i + \beta = \delta^{-1}(I_i - \Delta)$ into equation below:

$$\frac{dV_i}{dt} = -\left(\frac{1}{C_i R_i} + \sum_{j=1}^{N} \frac{1}{C_i R_{ij}}\right) V_i + \sum_{j=1}^{N} \frac{s_{ij}}{C_i R_{ij}} g(V_j) + i_i^{\text{in}}$$

That is

$$\frac{dI_i}{dt} = -\left(\frac{1}{C_i R_i} + \sum_{j=1}^{N} \frac{1}{C_i R_{ij}}\right)(I_i - \Delta) + \delta \sum_{j=1}^{N} \frac{s_{ij}}{C_i R_{ij}} \left[2\sigma(I_j) - 1\right] + \delta i_i^{\text{in}} \tag{5}$$

This should be the same as (3). So temporarily we get

$$\frac{1}{\tau_i} = \frac{1}{C_i R_i} + \sum_{j=1}^{N} \frac{1}{C_i R_{ij}} \tag{6}$$

and

$$w_{ij} = \frac{2\delta s_{ij}}{C_i R_{ij}} \tag{7}$$

(c) Plug (6) and (7) back into (5)

$$\frac{dI_i}{dt} = -\frac{1}{\tau_i}(I_i - \Delta) + \sum_{j=1}^{N} w_{ij}\sigma(I_j) - \sum_{j=1}^{N} \frac{w_{ij}}{2} + \delta i_i^{\text{in}}$$

Thus we get

$$I_i^{\text{inject}} = \frac{\Delta}{\tau_i} - \sum_{j=1}^{N} \frac{w_{ij}}{2} + \delta i_i^{\text{in}}$$

That is

$$i_i^{\text{in}} = \delta^{-1}\left(I_i^{\text{in}} - \frac{\Delta}{\tau_i} + \sum_{j=1}^{N} \frac{w_{ij}}{2}\right) \tag{8}$$

(d) From (7) and $C_i, R_{ij} > 0$, $s_{ij} \in \{1, -1\}$, we have (remember $\delta > 0$)

$$s_{ij} = \text{sgn}(w_{ij}) \tag{9}$$

$$C_i R_{ij} = \frac{2\delta}{|w_{ij}|} \tag{10}$$

From (6)

$$\frac{1}{C_i R_i} = \frac{1}{\tau_i} - \sum_{j=1}^{N} \frac{1}{C_i R_{ij}} = \frac{1}{\tau_i} - \frac{1}{2\delta}\sum_{j=1}^{N} |w_{ij}|$$

In order to ensure that $C_i R_i$ is positive, we can select δ big enough. Together with (10), we have (with C_i specified)

$$R_{ij} = \frac{2\delta}{|w_{ij}|\, C_i} \tag{11}$$

$$R_i = \frac{2\delta\tau_i}{\left(2\delta - \tau_i \sum_{j=1}^{N} |w_{ij}|\right) C_i} \tag{12}$$

2. Let

$$\mathbf{I} = \mathbf{W}\mathbf{V} + \mathbf{i}^{\text{in}}.$$

Then

$$\begin{aligned}
\frac{d\mathbf{I}}{dt} &= \mathbf{W}\frac{d\mathbf{V}}{dt} + \frac{d\mathbf{i}^{\text{in}}}{dt} \\
&= \mathbf{W}\left[-\frac{1}{\tau}\mathbf{V} + \sigma(\mathbf{I})\right] + \frac{d\mathbf{i}^{\text{in}}}{dt} \\
&= -\frac{1}{\tau}\mathbf{I} + \mathbf{W}\sigma(\mathbf{I}) + \left(\frac{1}{\tau}\mathbf{i}^{\text{in}} + \frac{d\mathbf{i}^{\text{in}}}{dt}\right).
\end{aligned}$$

That's the same form with (3), since we can take

$$\mathbf{I}^{\text{in}} = \frac{1}{\tau}\mathbf{i}^{\text{in}} + \frac{d\mathbf{i}^{\text{in}}}{dt}$$

2.3 Final Project Plan

1. **One experiment**. I recorded 'one' with preceding noise and post several experiments on it. One experiment done with cell #96 of Animal 1 (an 'on-set'-type β cell in area W with slow decay, Rostral location) has spikes lasting for ~ 0.9s. However, the organism did not show much response in my other experiements.

2. **Final project plan**. I will try to find the hypothesis of how *mus silicium* operates. I will 'start simple,' and 'grow fast,' and try to go through it again for checking. Below is the main parts of my plan.

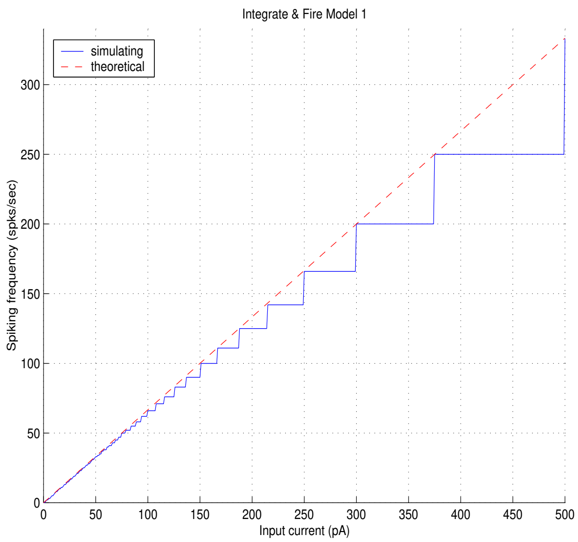
   (a) Testing cells of type α, β. Find out how input frequency affects the cells' output; Find the delay of the output; Use gradually changed tone to find out the sensitive range of cells; Try to find the interactions between α and β by using two tones and looking into the output of a γ cell.

   (b) Testing cells of type γ. Find out to a cell γ what is invarince for recognizing voice; Change frequency, filter some frequency or add some frequency to do that; Find out how temporal stimulus is recognized (I have not find a way to do this).

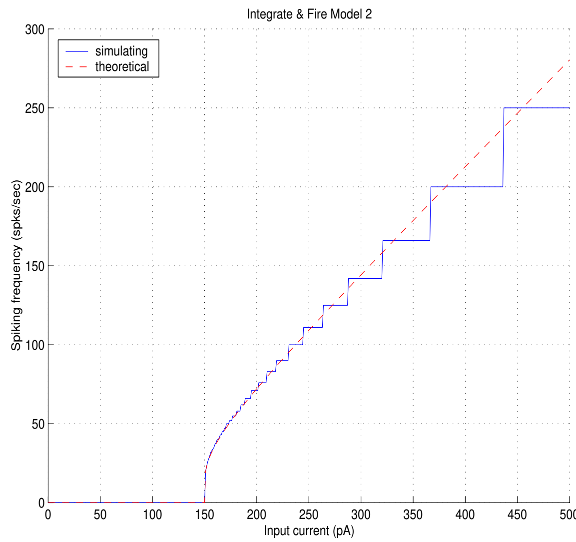
   (c) Overall testing: Try to find out the concrete constructure of the whole network.

3. **Schedule**.

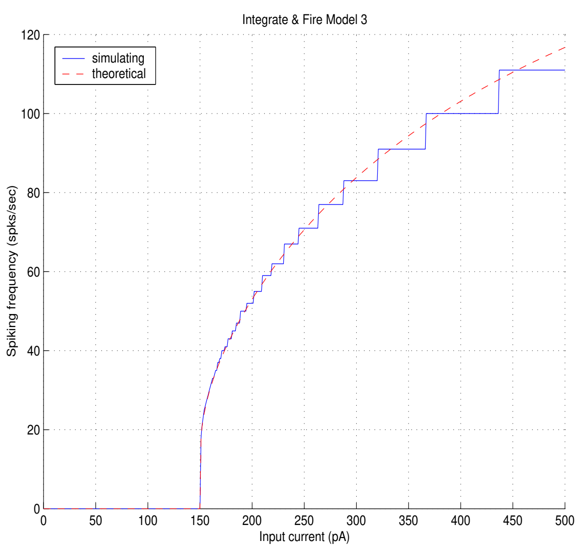
	Week#	Task (Find out ...)
1.	10/16~10/22	Functions of cells of type α, β
2.	10/23~10/29	Interactions between cells of type α, β (γ may be involved)
3.	10/30~11/05	Functions of cells of type γ
4.	11/06~11/12	Interactions within area *W*
5.	11/13~11/19	More experiments for overall test
6.	11/20~11/26	Paperwork
7.	11/27~12/03	Final checking and modification



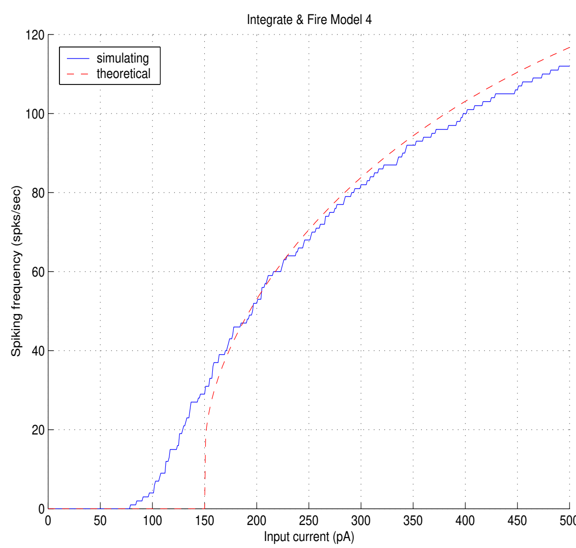
(a) Perfect I & F



(b) Leaky I & F



(c) Refactory I & F



(d) Refactory I & F with input noise

Figure 5: Simulating and theoretical spiking frequency vs input current (simulating step 1msec)