BEM 103 Introduction to Finance - Homework 4

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8.1 *States.* Let I^A and I^B be the prices of digital securities that pay off in states A and B, respectively.

(1) We have equations

$$\begin{aligned} 5 &= 10I^A + 10I^B \\ 4 &= 10I^A + 6I^B \end{aligned}$$

Thus $I^A = I^B = 0.25$, i.e., both prices are one quarter.

(2) The price of such security is $20I^A + 25I^B = 11.25$ dollars.

9.2 *Put lower bound.* Consider two strategies, A and B, in table below.

	Strategy	Cash flows Time t	Time T
A	sell put	P_t	$-\max(0, K - S_T)$
B	borrow $PV(K)$	$\frac{K}{(1+r)^{T-t}}$	$-K$
	buy stock	$-S_t$	S_T
	total	$\frac{K}{(1+r)^{T-t}} - S_t$	$-(K - S_T)$

We can see that strategy B always has equal or higher future cash flow. Thus B should cost more than A now, i.e.,

$$P_t \geq \frac{K}{(1+r)^{T-t}} - S_t.$$

9.4 *Put upper bound.* Consider two strategies, A and B, in table below.

	Strategy	Cash flows Time t	Time T
A	buy put	$-P_t$	$\max(0, K - S_T)$
B	save $PV(K)$	$-\frac{K}{(1+r)^{T-t}}$	K

Since $K \geq 0$ and $S_T \geq 0$, we know $\max(0, K - S_T) \leq K$. Thus strategy B should cost more than A now. That is

$$P_t \leq \frac{K}{(1+r)^{T-t}}.$$

9.5 *American put.* There are two reasons why exercising early is optimal. Firstly, now the option is in money. The earlier you exercise, the more you gain from the time value of the money. Second, the price of the underlying security changes according to its return rate, whose expectation is usually no less than the risk free interest rate, which is 5%. Thus the difference between the striking price and the security price tends to be smaller and smaller along the time. So exercising early will give you more return.

9.9 *BoA.* We can calculate the opportunity cost of exercising vs. not exercising for Ms. Johnson, then she will know she should not exercise.

If she exercises now, she loses the call option and her earning is $S_t - 55$, where 55 is the strike price, and S_t is the current BoA stock price. However, the call option itself has a value

$$C_t \geq S_t - 2 - \frac{55}{(1+10\%)^{6/12}} \approx S_t - 54.44 > S_t - 55.$$

Thus, keeping the option is a better choice.

14.4 *Calls, hedge.* Let I^u be the price of the digital option which pays \$1 when the stock price at the end of the year is \$150; and I^d be the prices of another digital option which pays \$1 when the stock price at the end of the year is \$75. The following two equations

$$\begin{aligned} 100 &= 150 I^u + 75 I^d, \\ 20 &= 50 I^u + 0 I^d, \end{aligned}$$

give $I^u = 6/15$ and $I^d = 8/15$. Thus the risk free interest rate r_f is

$$r_f = \frac{1}{I^u + I^d} - 1 = \frac{1}{14} \approx 7.14\%.$$

We can verify this by evaluating the cash flows of that 'perfect hedge portfolio':

Action		Now	1 year later	
			$S_T = 150$	$S_T = 75$
(1)	writing 3 calls	60	-150	0
(2)	buying 2 stocks	-200	300	150
(3)	borrowing \$140	140	$-140(1+r_f)$	$-140(1+r_f)$
Total		0	$150 - 140(1+r_f)$	

Since we pay/get nothing at present, the best we could expect 1 year later is that we also pay/get nothing. So

$$150 - 140(1+r_f) = 0 \quad \Longrightarrow \quad r_f = \frac{1}{14} \approx 7.14\%.$$

14.5 *Call.* Let p^u denote the state price probability that the underlying stock will be 120 five weeks later. We have for the current stock price,

$$96 \times (1+10\%)^{35/365} = p^u \cdot 120 + (1-p^u) \cdot 95.$$

So $p^u \approx 0.075256$. Thus we can price the call option as

$$\frac{1}{(1+10\%)^{35/365}} \left[p^u \cdot (120-112) \times 100 + (1-p^u) \cdot 0 \right] \approx 59.66.$$

14.6 *A*. First, the pricing of the stock gives

$$40 \times (1+8\%)^{1/12} = p^u \cdot 42 + (1-p^u) \cdot 38.$$

Solving the equation for p^u: $p^u \approx 0.56434$. The call option will earn \$3 if the stock price is 42 or nothing if the stock price is 38. Thus the option value is

$$\frac{1}{(1+8\%)^{1/12}} \left[p^u \cdot 3 + (1-p^u) \cdot 0\right] \approx 1.682.$$

15.3 *HAL*. $u = 1.25$, $d = 0.8$, and the per period (6 months) interest rate is

$$r = (1+20\%)^{1/2} - 1 \approx 9.5445\%.$$

So the state price probability on a period is

$$p^u = \frac{1+r-d}{u-d} \approx \frac{1 + 9.5445\% - 0.8}{1.25 - 0.8} \approx 0.6565.$$

(1) $S_0 = 9000$, and $K = 9000$. For a call option like that, we try to calculate its value at the end of the first period.

$$\begin{aligned} C_1^u &= \frac{1}{1+r}\left[p^u \cdot \max\left(0, u^2 S_0 - K\right) + (1-p^u) \cdot \max\left(0, udS_0 - K\right)\right] \\ &\approx 3034.16, \\ C_1^d &= \frac{1}{1+r}\left[p^u \cdot \max\left(0, udS_0 - K\right) + (1-p^u) \cdot \max\left(0, d^2 S_0 - K\right)\right] \\ &= 0. \end{aligned}$$

Since that is an option on this option, we will not buy the call option if the price is higher than its value. So the value of the “compound option” is

$$C_0 = \frac{1}{1+r}\left[p^u \cdot \max\left(0, C_1^u - 1500\right) + (1-p^u) \cdot \max\left(0, C_1^d - 1500\right)\right] \approx 919.49.$$

(2) If we do not have any choice, the value should be

$$C_0 = \frac{1}{1+r}\left[p^u \cdot \left(C_1^u - 1500\right) + (1-p^u) \cdot \left(C_1^d - 1500\right)\right] \approx 449.19.$$