BEM 103 Introduction to Finance - Homework 1

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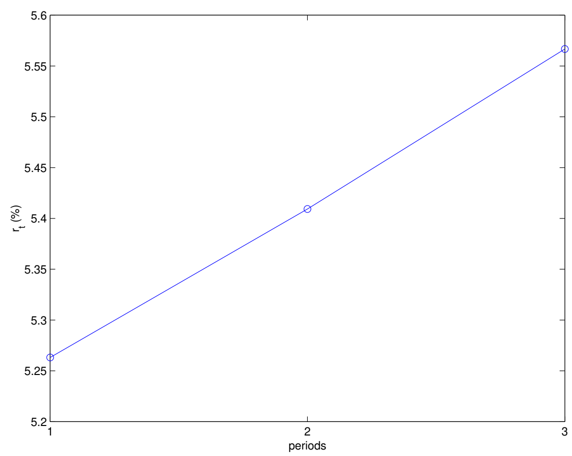
5.1 *Present Value.* From $r_t = \sqrt[t]{\frac{1}{P_t}} - 1$, we get

t	1	2	3
P_t	0.95	0.9	0.85
r_t	5.26%	5.41%	5.57%



The present value of cash flows $X_t = 100$ for $t = 1, 2, 3$ is

$$\mathrm{PV} = \sum_{t=1}^{3} P_t X_t = 270.$$

5.2 *Borrowing.* For a loan L from BankTwo, it will be

$$L \times \left(1 + \frac{10\%}{4}\right)^4 \approx 1.1038L$$

after 1 year. If it is from BankThree, it will be

$$L \times (1 + 10.5\%) = 1.105L.$$

So, BankTwo gives a better offer.

5.5 *Bonds.* We can set up equations of those bonds prices

$$\begin{aligned} 1000 \times (10\%P_1 + 10\%P_2 + P_2) &= 1043.29 \\ 1000 \times (20\%P_1 + 20\%P_2 + P_2) &= 1220.78 \\ 1000 \times (8\%P_1 + 8\%P_2 + 8\%P_3 + P_3) &= 995.09 \end{aligned}$$

Solve these equations, we get

$$P_1 \approx 0.9091, \quad P_2 \approx 0.8658, \quad P_3 \approx 0.7899.$$

From $r_t = \sqrt[t]{\frac{1}{P_t}} - 1$, we get

$$r_1 \approx 9.999\%, \quad r_2 \approx 7.471\%, \quad r_3 \approx 8.179\%.$$

5.6 *Stock.* $r = 10\%$. Today's stock price should be the sum of future payments. That is

$$\begin{aligned} P &= \frac{10 \times 1.1}{1+r} + \frac{10 \times 1.1^2}{(1+r)^2} + \sum_{t=3}^{\infty} \frac{10 \times 1.1^2 \times 1.02^{t-2}}{(1+r)^t} \\ &= 147.5\,. \end{aligned}$$

The just-paid dividend is not included when calculating P.

5.8 *Growing perpetuity.* For $r > g$,

$$\mathrm{PV} = \sum_{t=1}^{\infty} \frac{X_1(1+g)^{t-1}}{(1+r)^t} = \frac{X_1}{1+g} \sum_{t=1}^{\infty} \left(\frac{1+g}{1+r}\right)^t = \frac{X_1}{1+g} \sum_{t=1}^{\infty} \frac{1}{\left(1+\frac{r-g}{1+g}\right)^t} = \frac{X_1}{1+g} \cdot \frac{1}{\frac{r-g}{1+g}} = \frac{X_1}{r-g}.$$

5.9 *Annuity.* For $r > 0$, we have

$$\begin{aligned} \sum_{t=1}^{T} \frac{X}{(1+r)^t} &= \sum_{t=1}^{\infty} \frac{X}{(1+r)^t} - \sum_{t=T+1}^{\infty} \frac{X}{(1+r)^t} \\ &= \sum_{t=1}^{\infty} \frac{X}{(1+r)^t} - \frac{1}{(1+r)^T} \sum_{t=1}^{\infty} \frac{X}{(1+r)^t} \\ &= \frac{X}{r} - \frac{1}{(1+r)^T} \frac{X}{r} = X\left[\frac{1}{r} - \frac{1}{r}\frac{1}{(1+r)^T}\right]. \end{aligned}$$

5.12 *Jane.* If she invests X today, the return is $(1+5\%)^4 X$. Thus she needs to invest

$$X = \frac{55000}{(1+5\%)^4} \approx 45248.64\,.$$

If she invests X once a year, the return at the end of the fourth year is

$$X \sum_{t=1}^{4} (1+5\%)^t.$$

Thus the amount of investment should be

$$X = \frac{55000}{\sum_{t=1}^{4}(1+5\%)^t} \approx 12153.00\,.$$

6.3 *Machine*. Let's repeat the first option 4 times, the second option twice, and the third option once, to make them have the same life length. The PV of the costs for those options are

$$\sum_{t=0}^{3} \frac{47}{(1+10\%)^{2t}} \approx 144.47, \quad 90 + \frac{90}{(1+10\%)^4} \approx 151.47, \quad 300,$$

respectively. Thus the best option is to make some small repairs for a cost of 47.

6.5 *Project*. During the project life, the taxable income each year is

$$200 - 100 - \frac{240}{3} = 20,$$

so the tax is $20 \times 40\% = 8$, except the last year, which has more income.

$$\text{NPV} = -240 - 60 + \sum_{t=1}^{3} \frac{200-100-8}{(1+8\%)^t} + \frac{40 \times (1-40\%) + 60}{(1+8\%)^3} \approx 3.77.$$

6.6 *C&C*. Let X denote the units the C&C company produces per year. The accounting profits are

$$1.5X - 1500 - \frac{2000}{5} - 0.5X = X - 1900.$$

So the accounting break-even point is 1900 units per year. However, if $X > 1900$, the tax is

$$(X - 1900) \times 34\%$$

and the NPV of the machine is

$$-2000 + \sum_{t=1}^{5} \frac{X - 1500 - (X-1900) \times 34\%}{(1+16\%)^t} = \frac{(0.66X - 854)}{16\%} \left[1 - (1+16\%)^{-5}\right] - 2000,$$

and the PV break-even point is $X \approx 2219.42$, i.e., 2219 units per year.

6.7 *PillAdvent*. We calculate cash flows in real terms. Let M stand for million. For the headache-only medication, the income in year t is $5\text{M} \times (4 - 1.5) = 12.5\text{M}$, and the tax is

$$\left(12.5\text{M} - \frac{10.2\text{M}}{3 \times 1.05^t}\right) \times 34\%.$$

So the NPV over 3 years is

$$-10.2\text{M} + \sum_{t=1}^{3} \frac{12.5\text{M} - \left(12.5\text{M} - \frac{10.2\text{M}}{3 \times 1.05^t}\right) \times 34\%}{(1+13\%)^t} = 11.77\text{M}.$$

For the more general pill, the income in year t is $10\text{M} \times (4 - 1.7) = 23\text{M}$ and the tax is

$$\left(23\text{M} - \frac{12\text{M}}{3 \times 1.05^t}\right) \times 34\%.$$

Since in this case, we are able to sell the machine for 1M at the end of 3 years, the NPV is

$$-12\text{M} + \sum_{t=1}^{3} \frac{23\text{M} - \left(23\text{M} - \frac{12\text{M}}{3 \times 1.05^t}\right) \times 34\%}{(1+13\%)^t} + \frac{1\text{M} \times (1 - 34\%)}{(1+13\%)^3} \approx 27.23\text{M}.$$

So the company should produce the more general one.