ACM 113 **Problem Set 1*** April 11, 2001

Instructor: Niles A. Pierce
313 Firestone, x8086
niles@caltech.edu
Open office hours

TA: Andy Greenberg
210 Firestone, x4543
andyg@acm.caltech.edu
Office hours: 2:30-3:30pm M,F

1. Consider the unconstrained optimization problem

$$\min_{x \in \Re^2} f(x) = (x_1 - 2x_2)^2 + x_1^4.$$

Identify and classify any local or global minima.

2. Compute the gradient and Hessian of the Rosenbrock function

$$f(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2.$$

Show that $x^* = (1,1)^T$ is the only local minimizer and that it is strict. A look at the contours will suggest why this function is a popular test case for unconstrained optimization.

3. Consider the quadratic minimization problem

$$\min_{x \in \Re^n} f(x) = \frac{1}{2} x^T Q x - c^T x \tag{1}$$

with Q positive definite. Show that Newton's method converges to a strict minimizer of this problem in one step for an arbitrary initial guess. Why is this not surprising?

4. For the quadratic function given by (1) and a descent direction p_k, show that the line search subproblem

$$\min_{\alpha_k > 0} f(x_k + \alpha_k p_k)$$

has an exact solution for step length given by

$$\alpha_k = -\frac{p_k^T \nabla f_k}{p_k^T Q p_k}.$$

5. For $f(x)$ given by (1), program a steepest descent algorithm using an exact line search for $x \in \Re^2$. Construct three examples with condition numbers $\kappa \in \{1, 10, 100\}$. Estimate the convergence constant for each case and compare with the theoretical bound. Illustrate the convergence path by superimposing the steps on the objective contours. (Matlab is the preferred programming environment.)

*Late policy: Problem sets will be due at the end of lecture a week after they are handed out. For flexibility, you are allowed 5 extra days for all problem sets combined. Additional days will result in a 10% deduction per day on the late work.