Logistic regression - Outline

- The model
- Error measure
- Learning algorithm

How to minimize E_{in}

For logistic regression,

$$E_{\text{in}}(\mathbf{w}) = \frac{1}{N} \sum_{n=1}^{N} \ln\left(1 + e^{-y_n \mathbf{w}^{\mathsf{T}} \mathbf{x}_n}\right) \qquad \longleftarrow \textbf{iterative} \text{ solution}$$

Compare to linear regression:

$$E_{\text{in}}(\mathbf{w}) = \frac{1}{N} \sum_{n=1}^{N} (\mathbf{w}^{\mathsf{T}} \mathbf{x}_n - y_n)^2 \qquad \longleftarrow \text{closed-form solution}$$

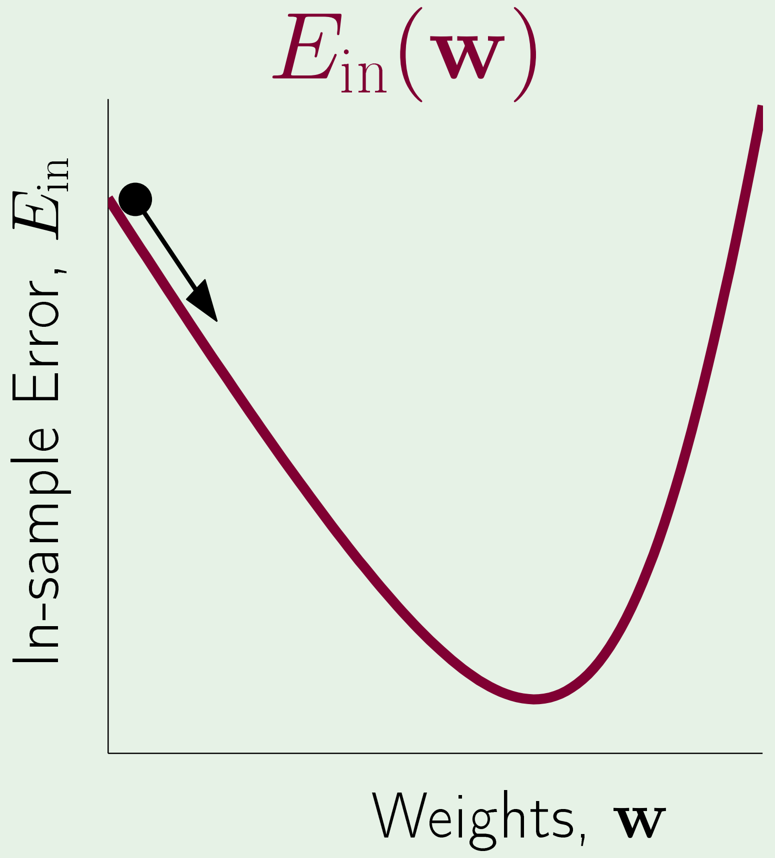
Iterative method: gradient descent

General method for nonlinear optimization

Start at $\mathbf{w}(0)$; take a step along steepest slope

Fixed step size: $\mathbf{w}(1) = \mathbf{w}(0) + \eta\, \hat{\mathbf{v}}$

What is the direction $\hat{\mathbf{v}}$?



Formula for the direction $\hat{\mathbf{v}}$

$$\Delta E_{\text{in}} = E_{\text{in}}(\mathbf{w}(0) + \eta\hat{\mathbf{v}}) - E_{\text{in}}(\mathbf{w}(0))$$

$$= \eta \nabla E_{\text{in}}(\mathbf{w}(0))^{\mathrm{T}} \hat{\mathbf{v}} + O(\eta^2)$$

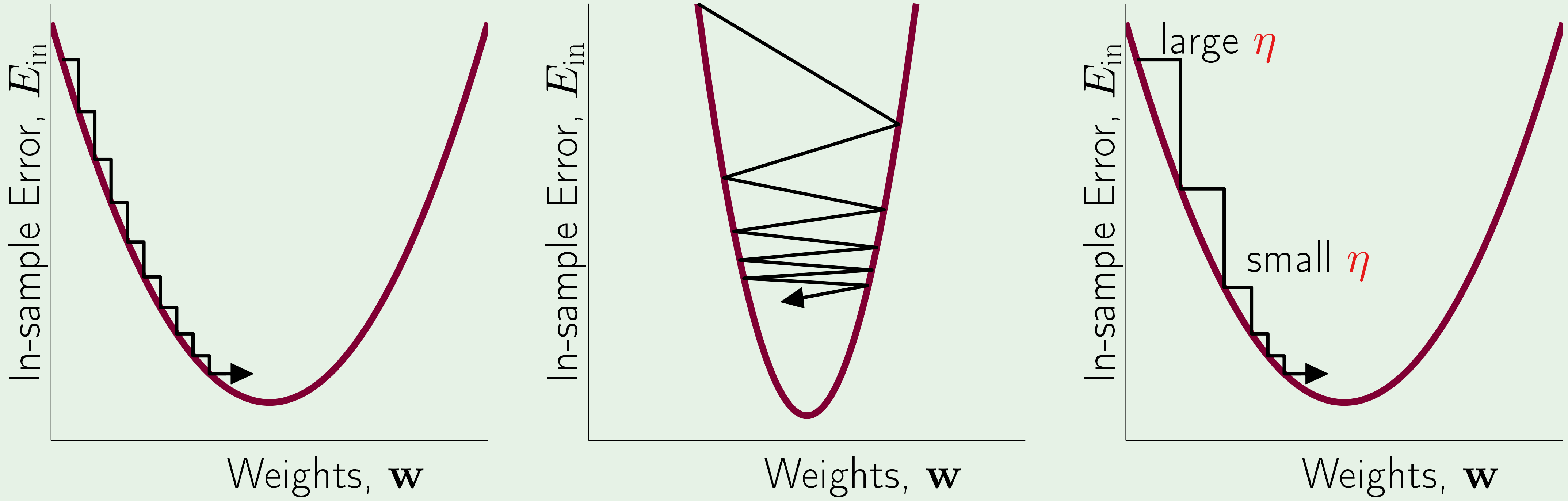
$$\geq -\eta \|\nabla E_{\text{in}}(\mathbf{w}(0))\|$$

Since $\hat{\mathbf{v}}$ is a unit vector,

$$\hat{\mathbf{v}} = -\frac{\nabla E_{\text{in}}(\mathbf{w}(0))}{\|\nabla E_{\text{in}}(\mathbf{w}(0))\|}$$

Fixed-size step?

How η affects the algorithm:



η too small — η too large — variable η – just right

η should increase with the slope

Easy implementation

Instead of

$$\Delta \mathbf{w} = \eta \, \hat{\mathbf{v}}$$

$$= - \eta \, \frac{\nabla E_{\text{in}}(\mathbf{w}(0))}{\|\nabla E_{\text{in}}(\mathbf{w}(0))\|}$$

Have

$$\Delta \mathbf{w} = - \eta \; \nabla E_{\text{in}}(\mathbf{w}(0))$$

Fixed **learning rate** η

Logistic regression algorithm

Initialize the weights at $t = 0$ to $\mathbf{w}(0)$

for $t = 0, 1, 2, \ldots$ **do**

  Compute the gradient

$$\nabla E_{\text{in}} = -\frac{1}{N} \sum_{n=1}^{N} \frac{y_n \mathbf{x}_n}{1 + e^{y_n \mathbf{w}^{\intercal}(t) \mathbf{x}_n}}$$

  Update the weights: $\mathbf{w}(t+1) = \mathbf{w}(t) - \eta \nabla E_{\text{in}}$

  Iterate to the next step until it is time to stop

Return the final weights $\mathbf{w}$