Back to the big picture

Remember this inequality?

$$\mathbb{P}\left[\left|E_{\text{in}} - E_{\text{out}}\right| > \epsilon\right] \leq 2M\, e^{-2\epsilon^2 N}$$

What happens if $m_{\mathcal{H}}(N)$ replaces M?

$m_{\mathcal{H}}(N)$ polynomial $\implies$ Good!

Just prove that $m_{\mathcal{H}}(N)$ is polynomial?

Outline

- From training to testing
- Illustrative examples
- Key notion: **break point**
- Puzzle

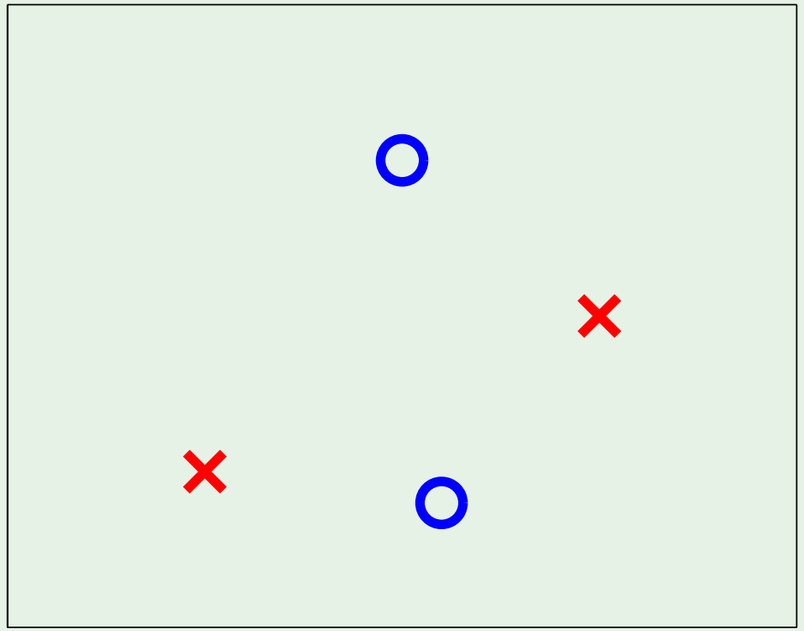
Break point of $\mathcal{H}$

Definition:

If no data set of size k can be shattered by $\mathcal{H}$, then k is a *break point* for $\mathcal{H}$

$$m_{\mathcal{H}}(k) \ < \ 2^k$$

For 2D perceptrons, $k = 4$

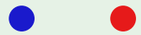


A bigger data set cannot be shattered either

Break point - the 3 examples

- Positive rays $m_{\mathcal{H}}(N) = N + 1$

  break point $k = 2$



- Positive intervals $m_{\mathcal{H}}(N) = \frac{1}{2}N^2 + \frac{1}{2}N + 1$

  break point $k = 3$

- Convex sets $m_{\mathcal{H}}(N) = 2^N$

  break point $k =$ '∞'

Main result

No break point $\implies$ $m_{\mathcal{H}}(N) = 2^N$

Any break point $\implies$ $m_{\mathcal{H}}(N)$ is **polynomial** in N