Outline
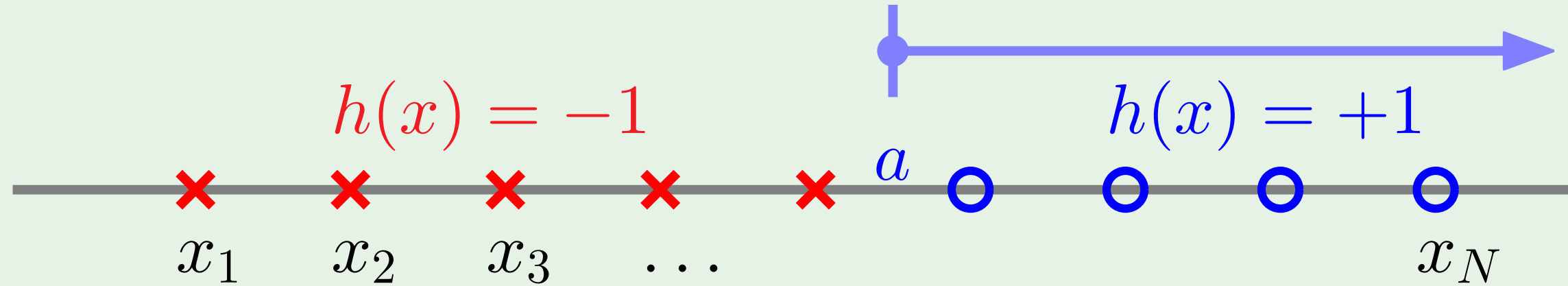
- From training to testing

- Illustrative examples

- Key notion: break point

- Puzzle

Example 1: positive rays
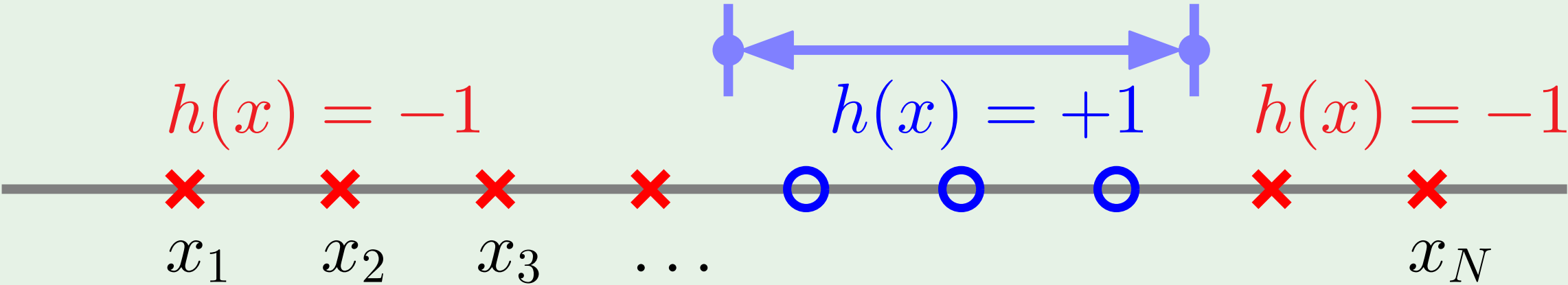

$h(x) = -1$ $h(x) = +1$

a

x_1 x_2 x_3 $\ldots$ x_N

$\mathcal{H}$ is set of $h\colon \mathbb{R} \to \{-1, +1\}$

$h(x) = \mathrm{sign}(x - a)$

$m_{\mathcal{H}}(N) = N + 1$

Example 2: positive intervals

$h(x) = -1$ $h(x) = +1$ $h(x) = -1$

x_1 x_2 x_3 $\ldots$ x_N

$\mathcal{H}$ is set of $h\colon \mathbb{R} \to \{-1, +1\}$

Place interval ends in two of $N + 1$ spots

$$m_{\mathcal{H}}(N) = \binom{N+1}{2} + 1 = \tfrac{1}{2}N^2 + \tfrac{1}{2}N + 1$$

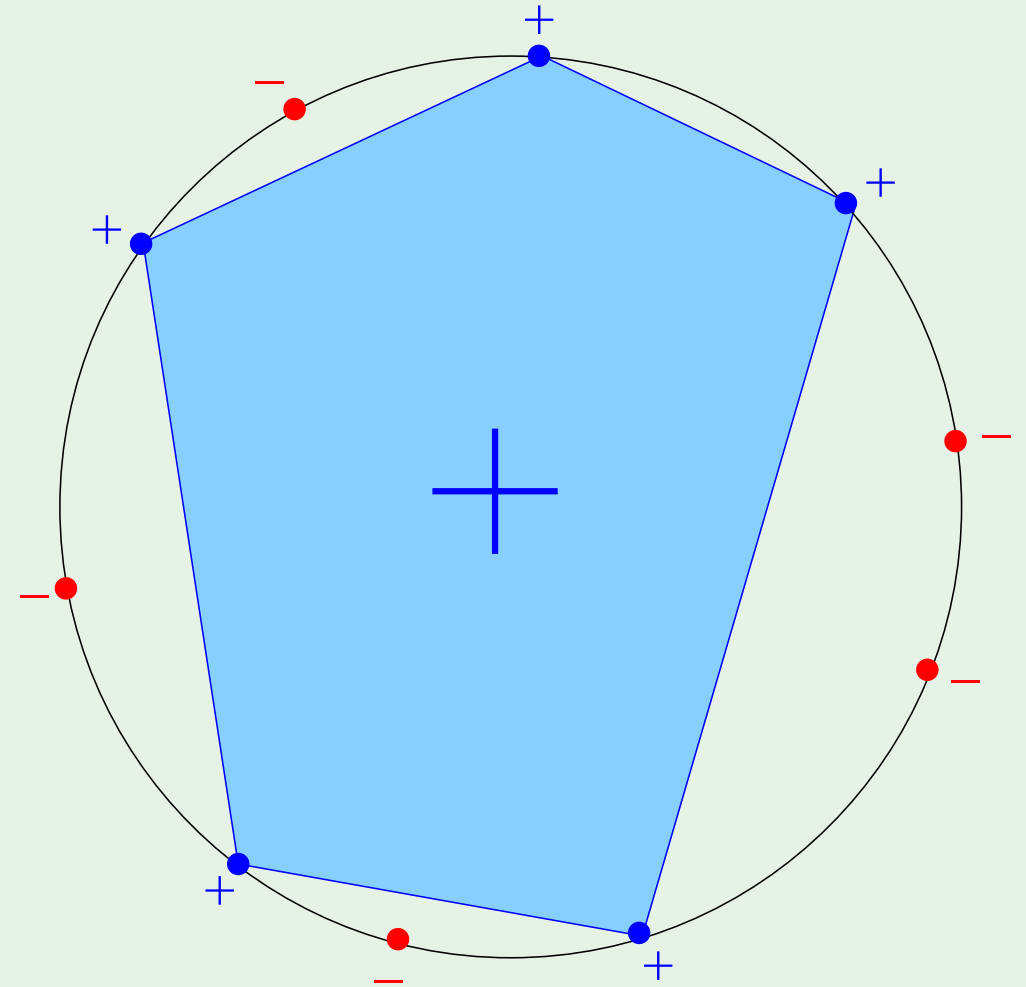# Example 3: convex sets

$\mathcal{H}$ is set of $h \colon \mathbb{R}^2 \to \{-1, +1\}$

$h(\mathbf{x}) = +1$ is convex

$m_{\mathcal{H}}(N) = 2^N$

The N points are 'shattered' by convex sets

The 3 growth functions

- $\mathcal{H}$ is positive rays:

$$m_{\mathcal{H}}(N) = N + 1$$

- $\mathcal{H}$ is positive intervals:

$$m_{\mathcal{H}}(N) = \tfrac{1}{2}N^2 + \tfrac{1}{2}N + 1$$

- $\mathcal{H}$ is convex sets:

$$m_{\mathcal{H}}(N) = 2^N$$