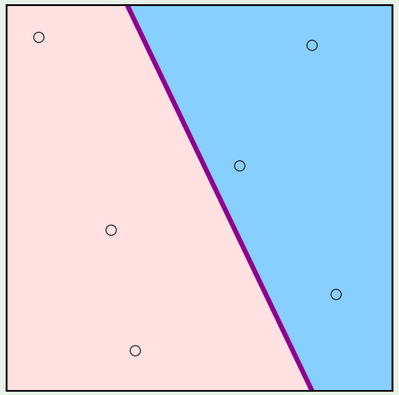
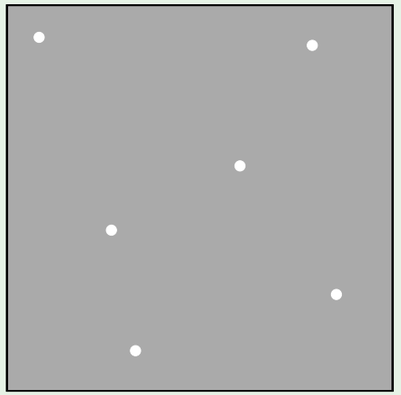
# What can we replace $M$ with?

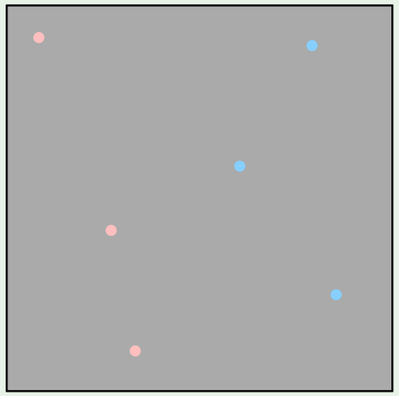
Instead of the whole input space,

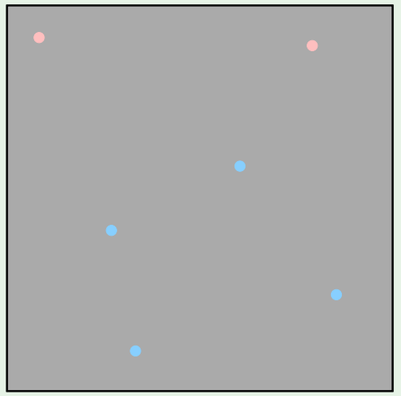
we consider a finite set of input points,

and count the number of *dichotomies*









# Dichotomies: mini-hypotheses

A hypothesis $h : \mathcal{X} \to \{-1, +1\}$

A dichotomy $h : \{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N\} \to \{-1, +1\}$

Number of hypotheses $|\mathcal{H}|$ can be infinite

Number of dichotomies $|\mathcal{H}(\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_N)|$ is at most $2^N$

Candidate for replacing $M$

# The growth function

The growth function counts the most dichotomies on any $N$ points

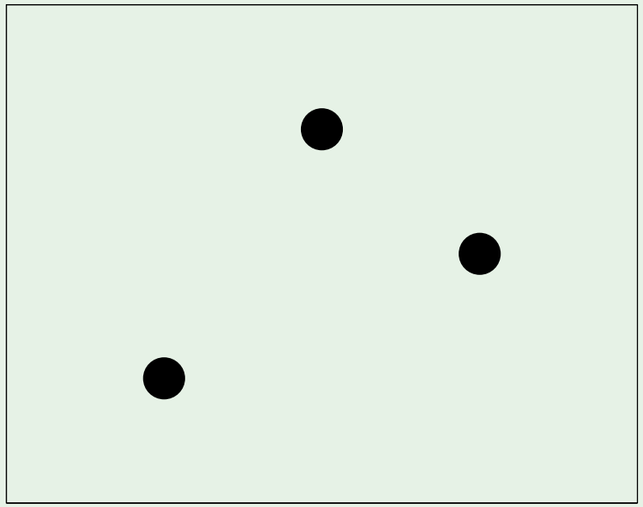
$$m_{\mathcal{H}}(N) = \max_{\mathbf{x}_1, \cdots, \mathbf{x}_N \in \mathcal{X}} |\mathcal{H}(\mathbf{x}_1, \cdots, \mathbf{x}_N)|$$

The growth function satisfies:

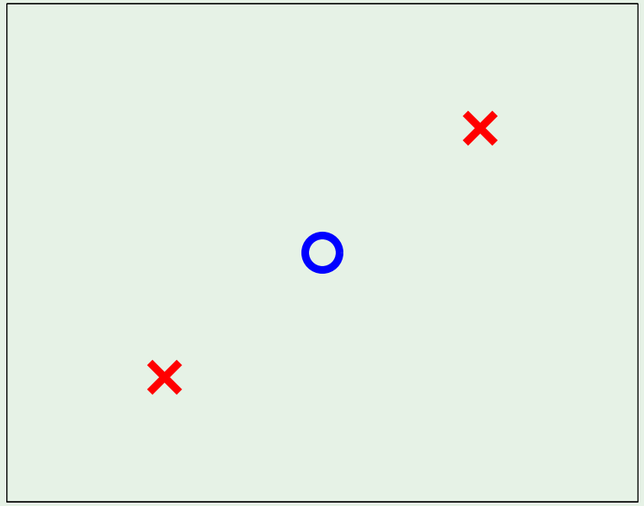
$$m_{\mathcal{H}}(N) \leq 2^N$$

Let's apply the definition.

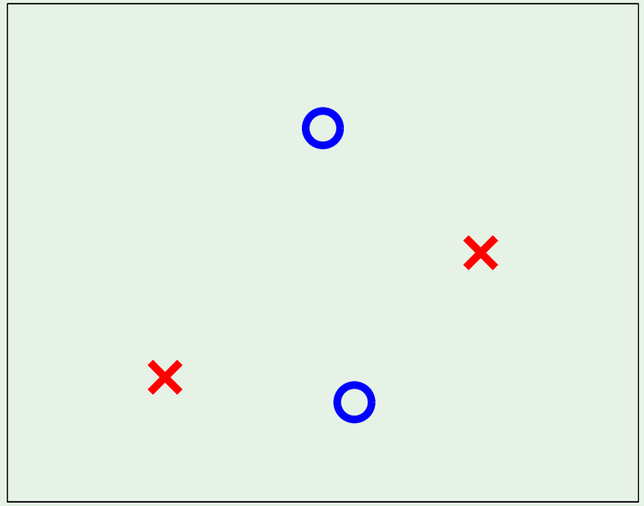
# Applying $m_{\mathcal{H}}(N)$ definition - perceptrons



$N = 3$



$N = 3$



$N = 4$

$m_{\mathcal{H}}(3) = 8$

$m_{\mathcal{H}}(4) = 14$