Error measures

What does "$h \approx f$" mean?

Error measure: $E(h, f)$

Almost always *pointwise definition*: $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right)$

Examples:

Squared error: $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right) = \left(h(\mathbf{x}) - f(\mathbf{x})\right)^2$

Binary error: $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right) = [\![h(\mathbf{x}) \neq f(\mathbf{x})]\!]$

From pointwise to overall

Overall error $E(h, f)$ = average of pointwise errors $\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right)$.

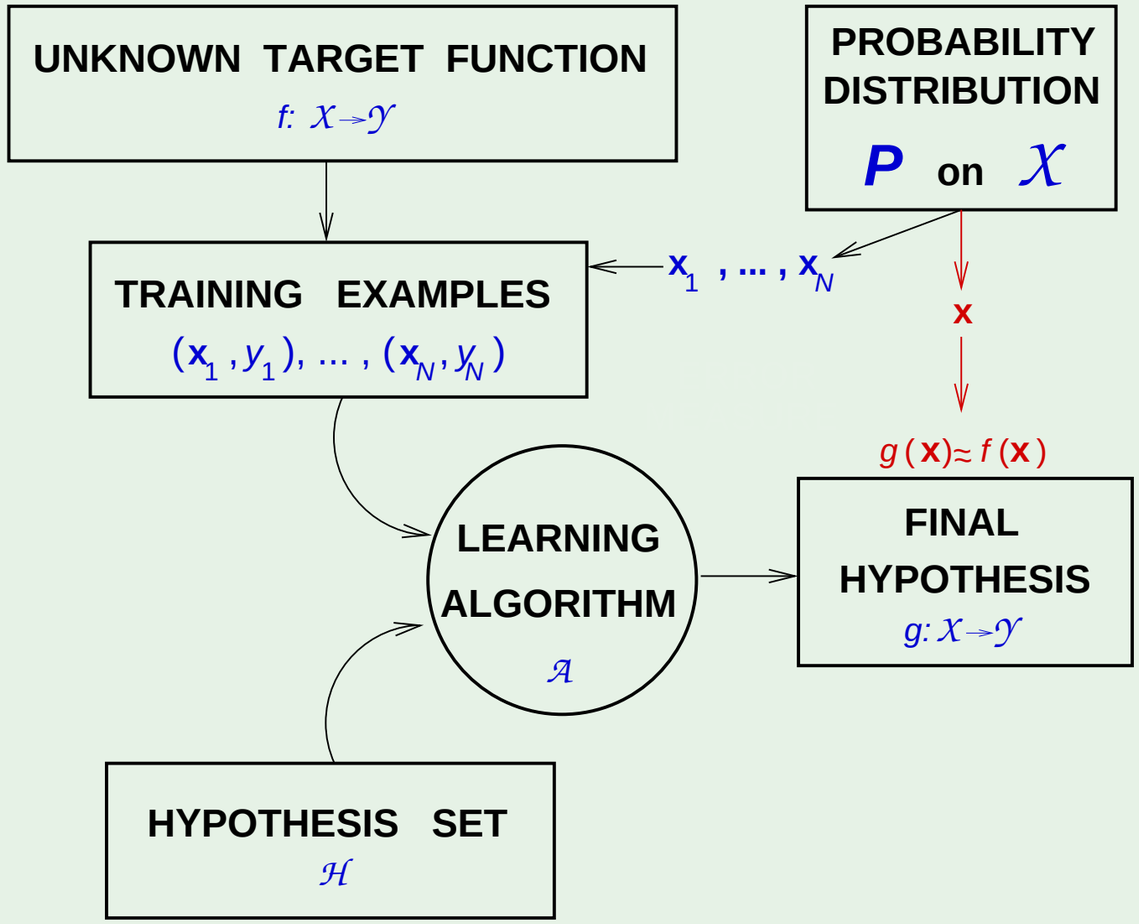
In-sample error:

$$E_{\text{in}}(h) = \frac{1}{N} \sum_{n=1}^{N} \mathrm{e}\left(h(\mathbf{x}_n), f(\mathbf{x}_n)\right)$$

Out-of-sample error:

$$E_{\text{out}}(h) = \mathbb{E}_{\mathbf{x}}\left[\mathrm{e}\left(h(\mathbf{x}), f(\mathbf{x})\right)\right]$$

The learning diagram - with pointwise error



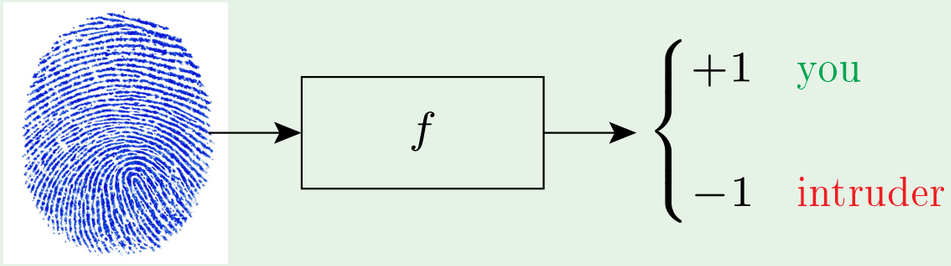
How to choose the error measure

Fingerprint verification:

Two types of error:

false accept and *false reject*

How do we penalize each type?



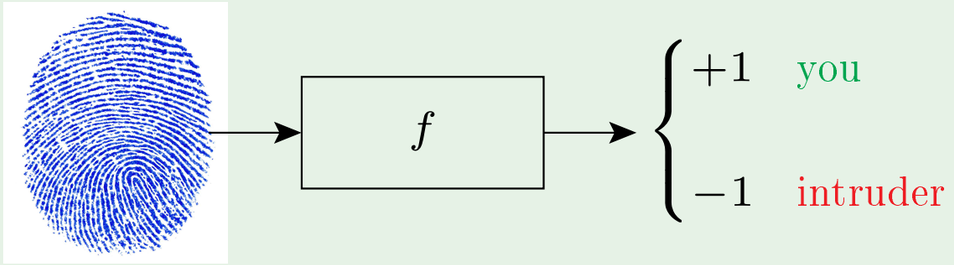
		f: $+1$	f: -1
h	$+1$	no error	*false accept*
	-1	*false reject*	no error

The error measure - for supermarkets

Supermarket verifies fingerprint for discounts

False reject is costly; customer gets annoyed!

False accept is minor; gave away a discount and intruder left their fingerprint ☺



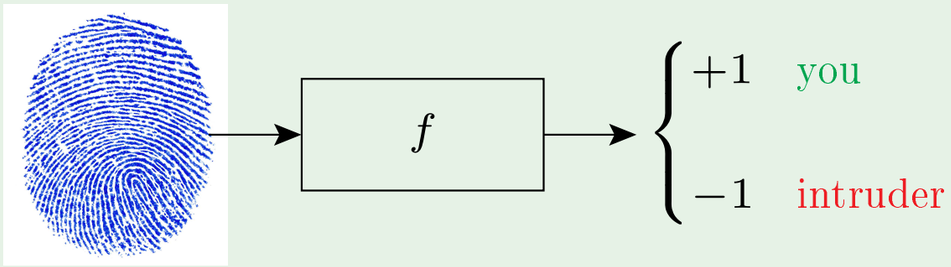
		f	
		$+1$	-1
h	$+1$	0	1
	-1	10	0

The error measure - for the CIA

CIA verifies fingerprint for security

False accept is a disaster!

False reject can be tolerated
Try again; you are an employee ☺



		f	
		$+1$	-1
h	$+1$	0	1000
	-1	1	0

Take-home lesson

The error measure should be specified by the user.

Not always possible. Alternatives:

Plausible measures: squared error $\equiv$ Gaussian noise

Friendly measures: closed-form solution, convex optimization

The learning diagram - with error measure