Coin analogy

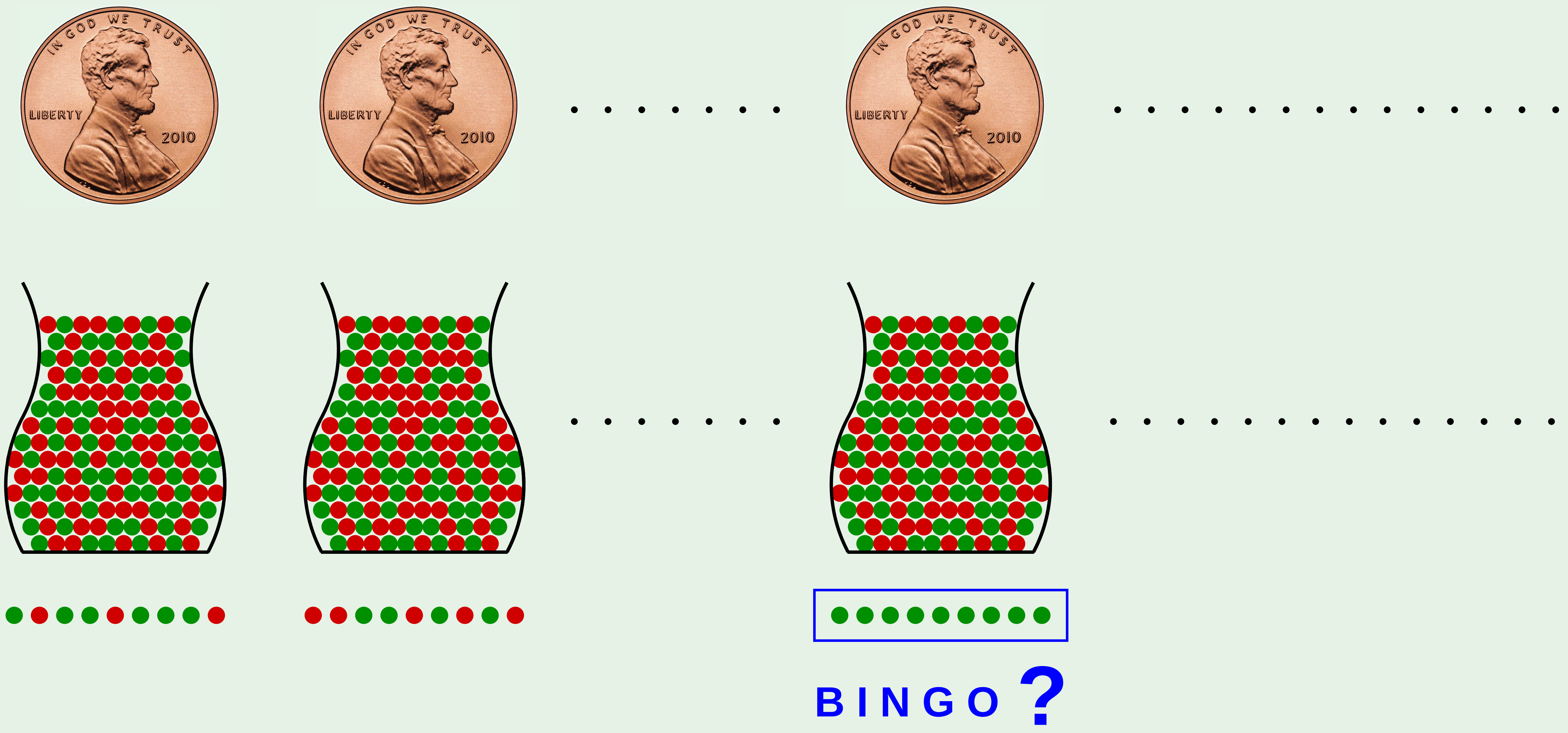
Question: If you toss a fair coin 10 times, what is the probability that you will get 10 heads?

Answer: $\approx 0.1\%$

Question: If you toss 1000 fair coins 10 times each, what is the probability that <u>some</u> coin will get 10 heads?

Answer: $\approx 63\%$

From coins to learning



A simple solution

$$
\begin{aligned}
\mathbb{P}[\ |E_{\text{in}}(g) - E_{\text{out}}(g)| > \epsilon\ ] \ \ \leq \ \ \mathbb{P}[\quad & |E_{\text{in}}(h_1) - E_{\text{out}}(h_1)| > \epsilon \\
& \textbf{or}\ |E_{\text{in}}(h_2) - E_{\text{out}}(h_2)| > \epsilon \\
& \cdots \\
& \textbf{or}\ |E_{\text{in}}(h_M) - E_{\text{out}}(h_M)| > \epsilon\ ] \\
\leq \ \ \sum_{m=1}^{M} \mathbb{P}\left[|E_{\text{in}}(h_m) - E_{\text{out}}(h_m)| > \epsilon\right] &
\end{aligned}
$$

The final verdict

$$\mathbb{P}[\,|E_{\text{in}}(g) - E_{\text{out}}(g)| > \epsilon\,] \;\leq\; \sum_{m=1}^{M} \mathbb{P}\,[|E_{\text{in}}(h_m) - E_{\text{out}}(h_m)| > \epsilon]$$

$$\leq\; \sum_{m=1}^{M} 2e^{-2\epsilon^2 N}$$

$$\mathbb{P}[|E_{\text{in}}(g) - E_{\text{out}}(g)| > \epsilon] \leq 2{\color{red}M}e^{-2\epsilon^2 N}$$