A simple hypothesis set - the 'perceptron'

For input $\mathbf{x} = (x_1, \cdots, x_d)$ 'attributes of a customer'

$$\text{Approve credit if} \quad \sum_{i=1}^{d} w_i x_i > \text{threshold},$$

$$\text{Deny credit if} \quad \sum_{i=1}^{d} w_i x_i < \text{threshold}.$$

This linear formula $h \in \mathcal{H}$ can be written as

$$h(\mathbf{x}) = \text{sign}\left(\left(\sum_{i=1}^{d} w_i x_i\right) - \text{threshold}\right)$$

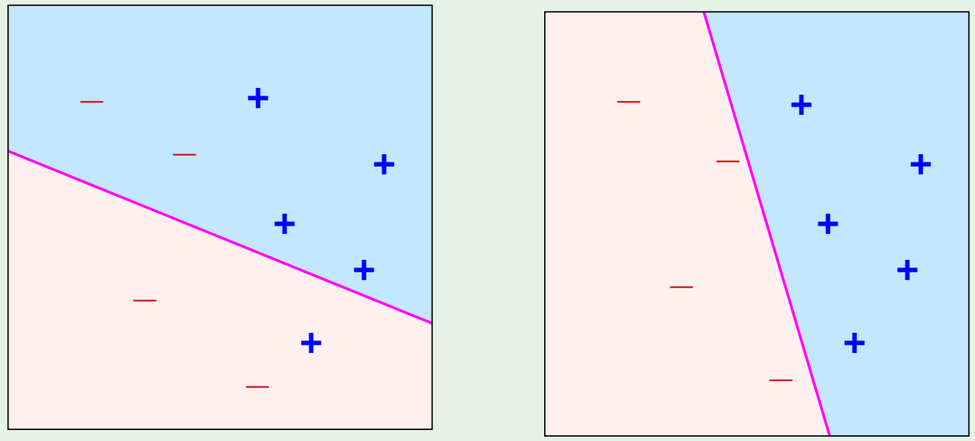
$$h(\mathbf{x}) = \text{sign}\left(\left(\sum_{i=1}^{d} w_i\, x_i\right) + \quad w_0 \quad\right)$$

Introduce an artificial coordinate $x_0 = 1$:

$$h(\mathbf{x}) = \text{sign}\left(\sum_{i=0}^{d} w_i\, x_i\right)$$

In vector form, the perceptron implements

$$h(\mathbf{x}) = \text{sign}(\mathbf{w}^{\mathsf{T}}\mathbf{x})$$



'linearly separable' data

A simple learning algorithm - PLA

The perceptron implements

$$h(\mathbf{x}) = \text{sign}(\mathbf{w}^{\mathsf{T}}\mathbf{x})$$

Given the training set:

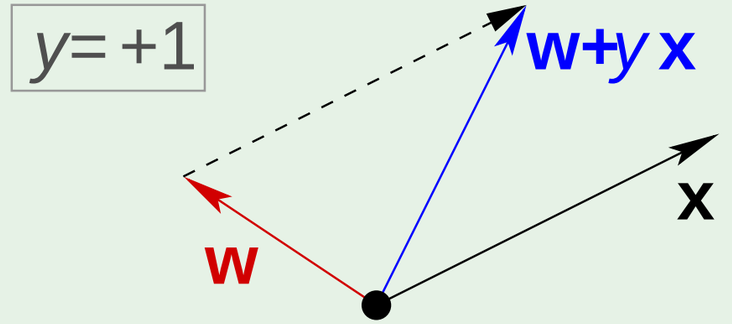
$$(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \cdots, (\mathbf{x}_N, y_N)$$

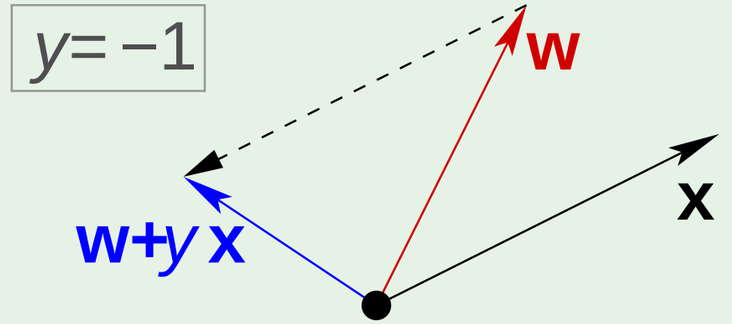
pick a misclassified point:

$$\text{sign}(\mathbf{w}^{\mathsf{T}}\mathbf{x}_n) \neq y_n$$

and update the weight vector:

$$\boxed{\mathbf{w} \leftarrow \mathbf{w} + y_n\mathbf{x}_n}$$





Iterations of PLA

- One iteration of the PLA:

$$\mathbf{w} \leftarrow \mathbf{w} + y\mathbf{x}$$

where $(\mathbf{x}, y)$ is a misclassified training point.

- At iteration $t = 1, 2, 3, \cdots$, pick a misclassified point from

$$(\mathbf{x}_1, y_1), (\mathbf{x}_2, y_2), \cdots, (\mathbf{x}_N, y_N)$$

and run a PLA iteration on it.

- That's it!